*STATISTICS IN TRANSITION new series, June 2026*
*Vol. 27, No. 2, pp. XX–XX*, https://doi.org/10.59139/stattrans-2026-xxxx
Received – xx.xx.xxxx; accepted – xx.xx.xxxx

# Comparison of two types of topological networks for the foreign exchange market: based on correlation coefficients and the concept of causality

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## Abstract

Topological networks make it possible to recognize structural properties of the currency market. Such networks can be constructed on the basis of the values of correlation coefficients between currency pairs, and the popular minimum spanning tree (MST) algorithm allows an understanding of significant relationships on the market. An alternative measure of distance, based on the concept of causality for time series, makes it possible to measure not only the strength of relationships between currency pairs but also the directionality of these relationships. There is even an equivalent of MST on a directed graph – minimum-cost arborescence (MCA). The purpose of this study is to compare correlation and causality networks built for the foreign exchange market. The networks were constructed in a stepwise manner for the most important world currencies in the period from 3rd Jan. 2020 to 18th Oct. 2024. The comparison was carried out using certain topological characteristics of the networks, such as density, average distance, diameter, centralization index and degrees of vertices. The study details the properties of both approaches.

**Key words:** topological networks, foreign exchange market, correlation, causality.

## 1. Introduction

Network analysis focuses on modeling various phenomena from the real world as a system. Network tools are used to study, for example, social networks, financial networks or computer networks. In each of these cases, a complex system is represented as a network with nodes playing the role of agents and edges representing the interconnections between nodes. By analyzing the connections between nodes in the topological structure of a network, hidden information about the network can be

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obtained. In the context of financial networks, such studies are conducted based on statistical relationships between series of returns.

Complex networks have been the subject of much research since the late 1990s. At first, small-world networks and scale-free networks were studied. The former refer to social networks in which the nodes and edges are people and their interactions (Watts and Strogatz, 1998). Here, individuals can be connected to each other with just a few random connections. The term scale-free network refers to a network whose degree distribution follows a power law (Barabási and Bonabeau, 2003). Both of these structures point to the unique features of the network. In particular, they emphasize that the real-world network does not have a random topology, but rather a centralized one with several hubs. The impact of the outbreak of certain events on network topology has also been studied. A network's topology is important for its resilience to external perturbations, such as failures or attacks (Albert et al., 2000; Cohen et al., 2000). For example, research has been conducted on the spread of diseases (Liu et al., 2004). Social problems such as traffic (e.g. Wu et al., 2008) and mobile communication (e.g. Hidalgo and Rodriguez-Sickert, 2008) have been solved using the network. The distribution of edges has led to consideration of the network community structure (Fortunato, 2010; Porter et al., 2009). Research has been conducted on the flow of information through nodes (Liu et al., 2016). The time-varying characteristics of networks were also analyzed (e.g. Palla et al., 2007).

Mantegna (1999) and Mantegna and Stanley (1999) introduced networks to the financial literature as a way to deal with the scale and number of complex relationships between economic agents. In this article, we attempt to model the structure of the global foreign exchange market. In the global foreign exchange market, the economic situation of each country and the interest rate policies implemented affect the exchange rates of neighbouring countries.

The purpose of the study is to compare two types of topological networks for the foreign exchange market: those based on correlation coefficients and those based on the concept of Granger causality for time series. We construct both correlation networks and causality networks, utilizing both undirected and directed edges. While correlation networks measure the strength of relationships between currency pairs, causality networks feature links that reflect significant directional effects from one currency to another. Beyond mere comparison, the study aims to demonstrate that causality networks act as a complementary framework to traditional correlation models. While the latter are effective for identifying overall market integration, the causality-based approach provides a deeper layer of 'informational richness' by uncovering the hidden directional architecture and lead-lag relationships of the FX market.

To achieve the general aim of this study, we formulate the following specific research questions:

Q1: do causality-based networks exhibit systematically different topological dynamics during crisis periods than correlation-based networks?

Q2: does the inclusion of directionality in the analysis change the identification of key 'driver' currencies in the global market compared to traditional undirected structures?

Q3: does the causality-based approach provide economically meaningful insights regarding market contagion and leadership that cannot be captured by analyzing correlations alone?

The comparative analysis of the two approaches is based on three criteria:

- Informational richness: The ability to identify lead-lag relationships and the direction of shock transmission.
- Sensitivity to market shifts: How quickly the network topology (e.g. density or diameter) responds to geopolitical shocks.
- Interpretability: The extent to which the resulting tree structure aligns with known economic dependencies (e.g. regional trade blocks).

Currency topological networks have already been studied, for example by McDonald et al. (2005), Ortega and Matesanz (2006), Naylor et al. (2007), Górski et al. (2008). Many researchers have focused on the structural evolution of the foreign exchange market during periods of crisis. Jang et al. (2011) and Feng and Wang (2010) noted that the correlation coefficient between currencies decreased during crises, while the length of the tree increased. Wang et al. (2012) studied the position of dominant world currencies. The correlation structure of the currency network was also studied by Kazemilari et al. (2018), Cao et al. (2020), Miśkiewicz (2021). An example of the research on the evolution of the currency network in the context of COVID-19 is the one conducted by Gupta and Chatterjee (2020). Another type of network is Granger causality networks, which are a common tool in mapping the human brain (Bullmore and Sporns, 2009), but they are also used in the financial literature, for example in the work of Billio et al. (2011), Výrost et al. (2015), Park et al. (2020), Jiang et al. (2022).

Although correlation-based MSTs and Granger causality networks have been individually applied to financial markets, there is a notable lack of comparative studies that evaluate their consistency and divergence during concurrent global shocks. Most existing literature treats these methods as alternatives rather than complements. Our study fills this gap by examining whether the 'causal architecture' of the FX market remains robust when the 'correlation architecture' shifts, thereby providing a more granular view of currency leadership that standard MST models cannot provide.

In the study, we analyzed changes in network topology based on time series of exchange rates of the most important world currencies from the period 03/01/2020 – 18/10/2024. Networks were constructed for the whole period as well as for 100-day rolling subsamples. Then the topological characteristics of the obtained graphs were analyzed.

The paper is organized as follows: Section 2 presents the two methods of analysis: correlation and causality networks, Section 3 describes the data used, Section 4 presents empirical results for currency market modeling obtained using both approaches, and Section 5 concludes.

## 2. Method of analysis

### 2.1. Correlation networks

An undirected graph is an ordered pair $G = (V, E)$, where $V$ is the set of vertices (nodes) and $E$ is the set of edges, which are two-element subsets of $V : E \subseteq \{\{u, v\}: u, v \in V\}$. A weighted graph is a graph in which each edge is assigned a weight that is some number (usually non-negative): $G = (V, E, w)$, where $w: E \to \mathbb{R}$.

One way to determine weights for edges in graphs (distances between nodes $X = (x_1, \dots, x_n)$ and $Y = (y_1, \dots, y_n)$) is to use Pearson's linear correlation coefficient:

$$\rho_p(X, Y) = \frac{\sum_{t=1}^{n}(x_t - \bar{x})(y_t - \bar{y})}{\sqrt{\sum_{t=1}^{n}(x_t - \bar{x})^2}\sqrt{\sum_{t=1}^{n}(y_t - \bar{y})^2}} \in [-1,1] \quad (1)$$

$$d_p(X, Y) = \sqrt{2[1 - \rho_p(X, Y)]} \in [0,2] \quad (2)$$

Links between objects in correlation networks can be defined in various ways. Pearson's linear correlation coefficient is preferred (Mantegna and Stanley (1999), Mizuno et al. (2006), Naylor et al. (2007), Jang et al. (2011)). Alternatively used measures of correlation are Spearman's rank correlation coefficient, Kendall's coefficient, partial correlation coefficient (Kenett et al., 2010; Basnarkov et al., 2019) or the coefficient of tail dependence estimated from the copula function (cf. Marti et al., 2021).

Starting from the correlation matrix between returns, the most important correlations can be extracted, resulting in a substantially sparser representation. Two approaches to filtering out the most important relationships are: (i) hierarchical methods and (ii) threshold methods. Among hierarchical methods, minimum spanning trees (MSTs) are the best known. MSTs, introduced by Kruskal (1956), compress information about network structure and simplify analysis by lowering the number of elements to be compared.

A spanning tree is a graph that is consistent and acyclic, i.e. there is a path between any two vertices and it is the only possible path between them. A minimum spanning tree *T* for a weighted undirected graph $G = (V, E, w)$ is a spanning tree (containing all vertices in the set *V*) for which the sum of the weights of all edges

$$w(T) = \sum_{(u,v)\in T} w(u, v) \tag{3}$$

is minimal (a minimum cost spanning tree). MST has $|V|$-1 edges, where $|V|$ is the number of vertices. To find MST, the Prim and Kruskal algorithms are used. Based on the topology of the network, a hierarchical structure can be built using the Girvan-Newman method, which uses the so-called edge betweenness.

### 2.2. Granger causality networks

A directed graph is an ordered pair $G = (V, A)$, where *V* is the set of vertices and *A* is the set of directed edges (arcs), which are two-element subsets of *V*, with edge $\{a, b\}$ understood to be directed from vertex $a$ to $b$.

In weighted directed networks, the weights may be determined by testing causality in the Granger sense (Granger 1969, 1980). By definition, a variable *X* is a cause of *Y* in the Granger sense if current values of *Y* can be predicted with greater accuracy using past values of *X* than without using them, with the remaining information unchanged. In the linear Granger causality test for pairs of variables, we estimate the equations of a VAR model with an equal number of lags for both variables, *k*, and apply a test of the joint significance of the lags of a given variable in the equation explaining the other variable:

$$y_t = \alpha_{10} + \sum_{j=1}^{k} \alpha_{1j} y_{t-j} + \sum_{j=1}^{k} \beta_{1j} x_{t-j} + \varepsilon_{1t} \tag{4}$$

$$x_t = \alpha_{20} + \sum_{j=1}^{k} \alpha_{2j} x_{t-j} + \sum_{j=1}^{k} \beta_{2j} y_{t-j} + \varepsilon_{2t} \tag{5}$$

$H_0\colon \beta_{11} = \beta_{12} = \cdots = \beta_{1k} = 0$ means there is no causal relationship in the Granger sense from *X* to *Y* (*X* is not the cause of *Y*). $H_0\colon \beta_{21} = \beta_{22} = \cdots = \beta_{2k} = 0$ means no causal dependence in the Granger sense from *Y* to *X* (*Y* is not the cause of *X*). In this paper, causality testing was performed using a wrapper in R: HDGC_VAR_all_I0 (Granger Causality Network in High Dimensional Stationary VARs) for stationary time series (I(0)), with lag $k$=1. The choice of the lag length was motivated by both economic and statistical considerations. In highly liquid foreign exchange markets, information is processed rapidly, and the primary interactions typically materialize within a single trading day. Statistically, we verified the optimal lag length for a representative subsample of pairs using the AIC criterion, which consistently pointed to $k$=1 or $k$=2 as the most appropriate structure. To maintain consistency and parsimony across all rolling windows and currency pairs in the network construction, a uniform lag of $k$=1 was adopted.

While foreign exchange markets often exhibit nonlinearities and tail dependencies, we employ the linear Granger causality framework as a robust baseline for identifying directional information flows in the conditional mean. This approach allows for a direct comparison with correlation-based networks and ensures the tractability of the resulting topological metrics. However, we acknowledge that this captures primarily short-term predictive relationships, and future research could extend this by employing non-linear or frequency-domain causality tests to capture higher-moment dependencies.

The intention of someone, given a directed graph $G = (V, A)$, would be to find a minimal spanning tree on it. However, this task is problematic. In MST, all vertices should be connected, and in a directed graph, not every node is reachable from every other node. Thus, directed graphs do not satisfy the requirement that all vertices are connected.

The equivalent of a minimum spanning tree on a directed graph is a spanning arborescence of minimum weight (MSA) or otherwise optimum branching. An *r*-arborescence of a graph $G$ is a directed tree $T$ that contains a directed path from a specified node $r$ to each node of a subset $V'$ of the set $V \setminus \{r\}$. The node $r$ is called the root of the arborescence. The algorithm for finding MSA is the Chu-Liu/Edmonds algorithm.

The construction of undirected and directed graphs in R is possible thanks to functions available in the *igraph* (the *mst* function allows the creation of MSTs) and *optrees* (*msArborEdmonds* allows building of MSAs) packages.

### 2.3. Topological characteristics of networks

Topological network indexes were used to study the dynamically changing structure of constructed networks. The following measures were applied at the level of the entire graph:

- density – the ratio of the number of edges ($|E|$) to the largest possible number of edges. Low density indicates greater independence between agents.

$$density(G) = \frac{|E|}{|V|(|V|-1)/2} \tag{6}$$

- mean distance (*apl*) – an average distance between all pairs of nodes in the graph. A low *apl* value indicates the efficiency of information flow in the network and indicates a structure susceptible to infection by negative events.

$$apl(G) = \frac{1}{|V|(|V|-1)} \sum_{i \neq j} d(v_i, v_j) \tag{7}$$

$|V|$ – the number of vertices, $d(v_i, v_j)$ – the length of the shortest path between nodes $i$ and $j$.

- diameter – the length of the longest shortest path between two nodes. A smaller diameter favors the transmission of information.

- degree centrality - the graph's centralization index regarding the number of links that nodes have. The higher it is, the higher the risk that nodes will intercept everything that flows through the network.

At the node level, the following indicators were used:

- degree – the degree of a vertex, i.e. the number of edges entering and leaving the vertex,
- in-degree – the input degree of a vertex, i.e. the number of edges entering the vertex,
- out-degree - the output degree of a vertex, i.e. the number of edges leaving the vertex.

To calculate the values of the above metrics, functions available in R within the *igraph* package were used.

## 3. Data used in the study

Daily data for the exchange rates of 15 currencies against the New Zealand Dollar (X/NZD) for the period 3/01/2020 – 18/10/2024 were obtained from https://stooq.com. A list of the currencies studied, along with their abbreviations, is shown in Table 1.

**Table 1.** List of the currencies studied

| Abbreviation | Currency name | Abbreviation | Currency name |
|---|---|---|---|
| CAD | Canadian Dollar | KRW | South Korean Won |
| CHF | Swiss Franc | NOK | Norwegian Krone |
| CNY | Chinese Yuan | PLN | Polish Zloty |
| EUR | Euro | RUB | Russian Ruble |
| GBP | Pound Sterling | SEK | Swedish Krona |
| HKD | Hong Kong Dollar | SGD | Singapore Dollar |
| ILS | New Israeli Shekel | USD | US Dollar |
| JPY | Japanese Yen | | |

*Source: authors' work.*

The choice of base currency (numéraire) is a problem for which there is no standard solution. Currencies are valued against each other, so there is no independent numéraire. Different choices will yield different results. In this study, we chose the New Zealand Dollar (NZD) as the reference currency. This decision is grounded in the methodological framework proposed by Kwapień et al. (2009), who demonstrated that using a dominant global currency (like the USD or EUR) as a numéraire 'leads to a star-like MST structure (...) and does not represent the true relationships among the remaining currencies.' By choosing a liquid yet peripherally located currency that is not a 'driver' for a major regional trade bloc, we minimize the risk of artificial centralization and common-factor-induced distortions.

Furthermore, the stability of the network's topological hierarchy under this approach is supported by our previous research (Andrzejak et al., 2024), which specifically investigated the impact of different distance measures and reference frames on the consistency of currency networks. Our findings in that study confirmed that while the numéraire affects absolute correlation levels, the relative hierarchical positions of major currencies (such as the centrality of the SGD or EUR) remain robust. Consequently, the observed changes in network topology reported in this paper reflect genuine structural shifts in the market rather than artifacts of the data transformation. Additionally, the use of causality-based networks provides a further layer of robustness, as these measures are inherently less sensitive to the common-factor bias introduced by the numéraire compared to traditional correlation-based MSTs.

On the other hand, it is known that the choice of base currency strongly affects Pearson's linear correlation, while partial correlations would be invariant in this aspect (Basnarkov et al., 2019).

Prior to the analysis, all exchange rate series were transformed into logarithmic returns to ensure stationarity according to the formula:

$$R(t) = log\left(\frac{P(t+1)}{P(t)}\right) = log\, P(t+1) - log\, P(t) \tag{8}$$

Augmented Dickey-Fuller (ADF) tests were performed for all series across all sub-periods, confirming that the variables are I(0) at the 5% significance level. Furthermore, by employing a rolling window approach, we explicitly account for local non-stationarity and potential regime changes triggered by the pandemic and the conflict in Ukraine, allowing the network topology to evolve as market conditions shift.

## 4. Empirical results

In the constructed graphs, vertices represent exchange rates, while edge weights reflect the distances between series of returns for exchange rates. The edges in MSTs are undirected, while in Granger causality graphs they are directed. The networks were constructed for the entire period 3/01/2020 – 18/10/2024 as well as for 100-day rolling subsamples with a two-week step. 115 minimum spanning trees and 115 Granger causality graphs with significant edges were built for 115 sub-periods with 15 vertices each.

The MSTs were built on the basis of correlation graphs, in which the distance between nodes was calculated based on the values of the correlation coefficients for the return series (formula (2)). The Granger causality test led to the construction of directed graphs, in which the edge weights were the p-values from the causality test for currency pairs. The full Granger graphs in a further stage of the analysis were reduced to graphs in which the edge weights satisfy the condition p-value<0.05 (rejecting $H_0$ of non-causality in the Granger test).

In the construction of Granger causality networks, a significance level of α=0.05 was applied to identify valid edges. We acknowledge the potential risk of Type I errors associated with multiple testing. However, the use of more restrictive corrections, such as the Bonferroni correction, often leads to excessive sparsity in financial networks, potentially masking meaningful structural information. To mitigate this, in the remainder of our study, we focus on the evolution of topological metrics over time rather than the existence of individual edges.

Figure 1 presents graphs for three arbitrarily selected sub-periods: VIII–XII.2020, II–VI.2021, XI.2021–III.2022. The first is a period of uncertainty related to the COVID-19 pandemic and the US presidential election. The second is a period of monetary easing with numerous social transfers and money printing around the world. The third includes Russia's invasion of Ukraine and the energy crisis. The events observed in the world have significantly affected the structure of the currency network. In particular, it can be seen that greater uncertainty in the markets has resulted in lower density in the causality graph.

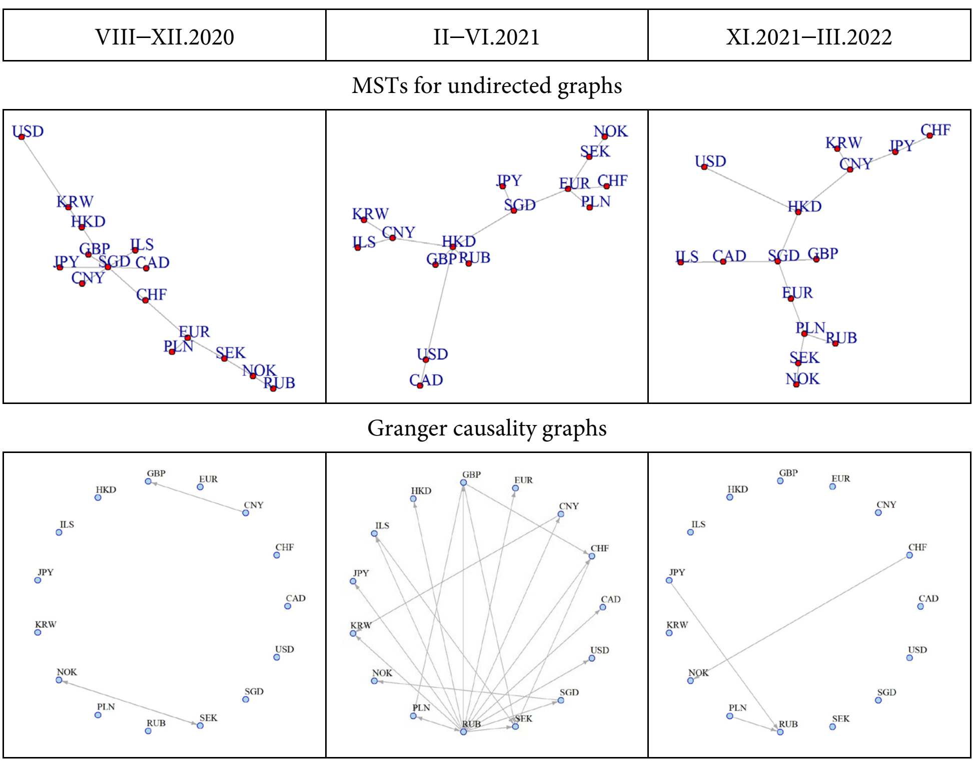


**Figure 1.** Structure of the currency network in three selected sub-periods: VIII–XII.2020, II–VI.2021, XI.2021–III.2022

*Source: authors' work based on data from stooq.com (accessed October 21, 2024).*

For all 230 networks obtained using the rolling window technique, the values of such topological characteristics as density, mean distance, diameter, and degree centrality were determined and presented in graphs (see Figure 2 for MSTs and Figure 3 for Granger networks).

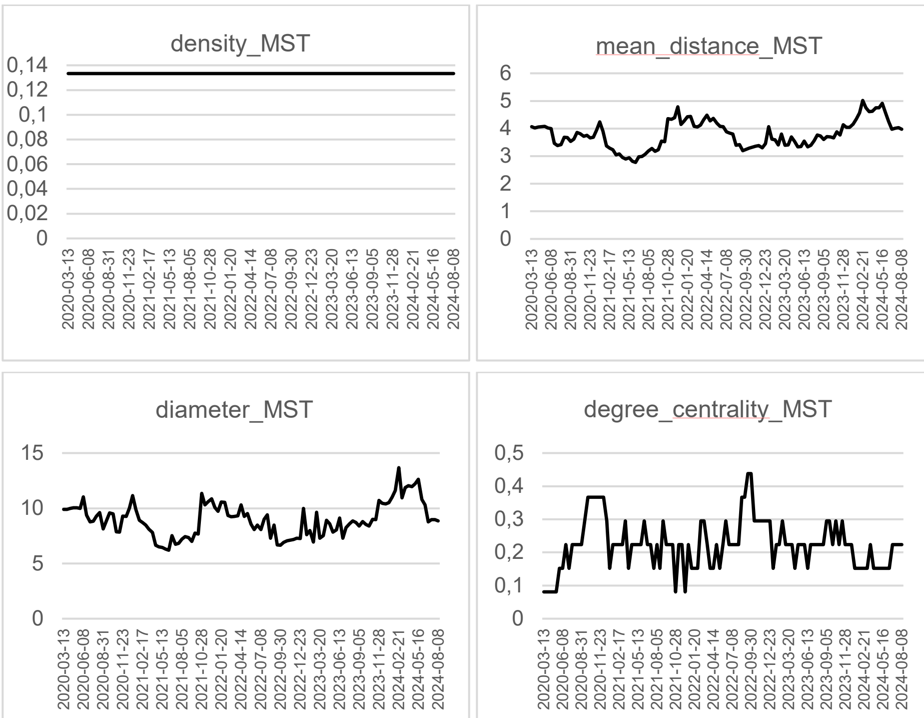


**Figure 2.** Selected characteristics of constructed minimum spanning trees (MST)

*Source: authors' work based on data from stooq.com (accessed October 21, 2024).*

It should be noted that MSTs for the currency market are characterized by a constant level of density (upper left panel of Figure 2). This follows from equation (6), where for MSTs we have $|E| = |V| - 1$. The remaining measures indicate different topological properties during the period considered. Lower average distance between nodes and lower graph diameter characterized the year 2021. Higher values for these measures occurred in 2020 (pandemic outbreak), 2022 (Russian invasion of Ukraine), and 2024. These years were also marked by lower graph centralization. It can be argued that during the difficult events of recent years, currencies were less interconnected (less correlated). Other authors have also noted that the correlation between currencies decreases during crises, while the length of the tree increases; see Jang et al. (2011), Feng and Wang (2010).

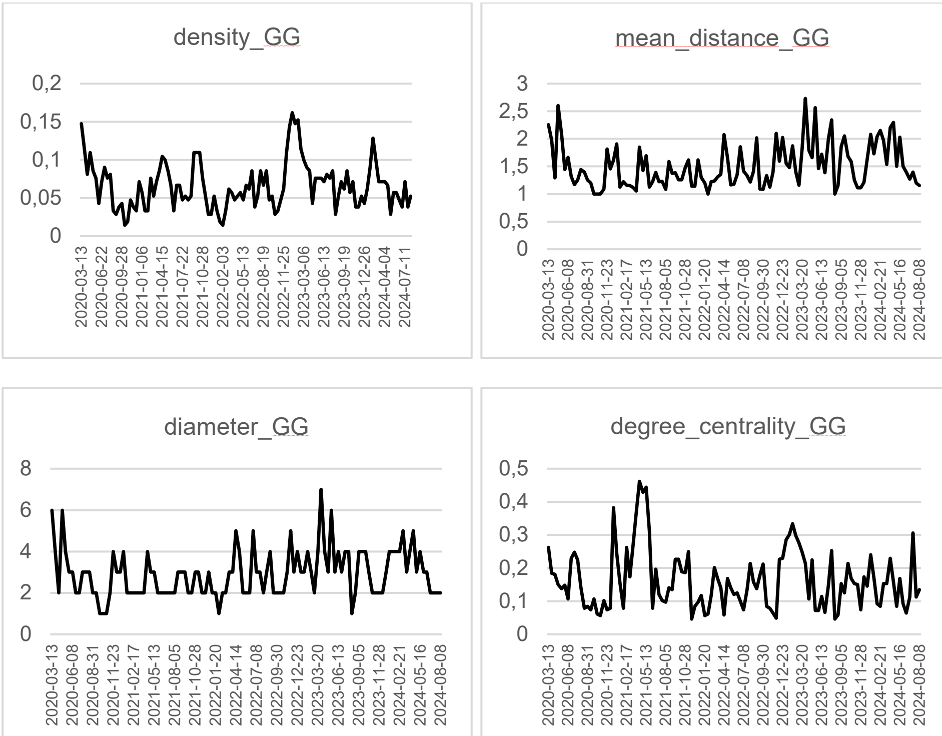


**Figure 3.** Selected characteristics of constructed Granger causality graphs (GG)

*Source: authors' work based on data from stooq.com (accessed October 21, 2024).*

In the case of Granger causality networks, their density is variable. The upper left panel of Figure 3 shows its successive decline caused by the freezing of economies and the fall in GDP in late 2020 (pandemic effect) and in February 2022 (Russian invasion). In contrast, peaks in density are visible in the spring and fall of 2021 (money transfers and commodity boom) and at the beginning of 2023 (growth of the money supply in the US, inflation). The density of the causality network appears to be lower during crisis periods. As a result of crises, the number of connections in networks decreases. A similar result was obtained by Park et al. (2020).

The values of the other network characteristics for MSTs and Granger graphs (GGs) are weakly correlated with each other. Table 2 presents Pearson correlations between the characteristics for the constructed graphs.

Analyzed separately, in both MSTs and GGs, the mean distance and longest shortest path (diameter) are strongly correlated. There is a positive correlation between density and mean distance (also between density and diameter, and centralization index) in Granger graphs.

**Table 2.** Pearson correlations between characteristics for graphs

| Specification | density_ MST | mean_distance_ MST | diameter_ MST | degree_centrality_ MST | density_ GG | mean_distance_ GG | diameter_ GG | degree_centrality_ GG |
|---|---|---|---|---|---|---|---|---|
| Density_MST | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Mean_distance_MST | | 1.00 | 0.90*** | -0.46*** | -0.13 | 0.20** | 0.16* | -0.30*** |
| Diameter_MST | | | 1.00 | -0.51*** | -0.11 | 0.23** | 0.16* | -0.23** |
| Degree_centrality_MST | | | | 1.00 | -0.23** | -0.30*** | -0.29*** | -0.06 |
| Density_GG | | | | | 1.00 | 0.45*** | 0.45*** | 0.61*** |
| Mean_distance_GG | | | | | | 1.00 | 0.94*** | 0.14 |
| Diameter_GG | | | | | | | 1.00 | 0.12 |
| Degree_centrality_GG | | | | | | | | 1.00 |

*, **, *** denote statistical significance at the 10%, 5%, and 1% level, respectively

*Source: authors' own calculations based on data from stooq.com (accessed October 21, 2024).*

MSTs are characterized by a negative correlation between graph centralization and mean distance (as well as the longest shortest path). This is because the lack of connection between nodes lengthens the path. In causality graphs, there is no significant correlation between graph centralization and average distance (and also the diameter). Interesting and similar considerations on this topic can be found in Park et al. (2020).

There seems to be no relationship between degree centralization in MSTs and Granger graphs (the correlation coefficient is close to zero). Low density in GGs harmonizes with a high centralization index in MSTs. The relationship between density in GGs and mean distance in MSTs is also negative but not statistically significant.

From an economic perspective, the shift in the causality-based networks structure during the Ukraine war suggests that causality is not just a statistical artifact but reflects a reconfiguration of safe-haven flows. Unlike MST, which only shows that currencies move together, our causality analysis identifies which currencies triggered the movement, providing a 'early warning' signal for systemic risk.

It is worth looking at the degrees of vertices in the constructed graphs. Table 3 contains the mean vertex degrees and the estimates along with statistical significance information for the coefficient $\beta$ in the $degree_t = \alpha+\beta \cdot t + \varepsilon_t$ model.

Bold values refer to vertices with the highest degree (SGD, EUR, HKD in MSTs; RUB, CAD, SGD, and JPY in GGs). The causality networks allowed us to obtain valuable additional information about the in/out-degrees of the vertices. Values in italics are for the most "influential" – CHF, ILS, RUB, SGD – and the least "susceptible to influence" – CHF, EUR, ILS – vertices. On the other hand, the most "influence-prone" and least "influential" currencies turned out to be NOK, CAD, GBP, JPY, and PLN. The estimated trend models identified the weakening influence of CNY and USD, the strengthening position of GBP and NOK, and the weakening position of JPY in the network.

**Table 3.** Degrees of vertices in constructed graphs

| Speci-fication | Degree MST | | Degree GG | | In-degree GG | | Out-degree GG | |
|---|---|---|---|---|---|---|---|---|
| | mean | trend | mean | trend | mean | trend | mean | trend |
| CAD | 1.28 | -0.002 | **2.20** | 0.007 | 1.19 | 0.006 | 1.01 | 0.001 |
| CHF | 1.53 | -0.005** | 1.83 | -0.003 | *0.64* | -0.001 | *1.18* | -0.002 |
| CNY | 2.14 | 0.002 | 1.92 | -0.010** | 1.08 | 0.006* | 0.84 | -0.016*** |
| EUR | **3.85** | 0.011*** | 1.54 | 0.008 | *0.48* | -0.003 | 1.06 | 0.011** |
| GBP | 1.14 | 0.003*** | 1.48 | -0.004 | 0.84 | -0.008*** | 0.63 | 0.004 |
| HKD | **2.64** | -0.013*** | 1.88 | -0.002 | 0.87 | 0.001 | 1.01 | -0.003 |
| ILS | 1.03 | -0.001*** | 1.74 | 0.010 | *0.63* | -0.001 | *1.11* | 0.011* |
| JPY | 1.17 | -0.005*** | **2.10** | 0.018*** | 1.14 | 0.009*** | 0.96 | 0.009** |
| KRW | 1.03 | 0.000 | 1.93 | 0.004 | 1.10 | 0.006* | 0.83 | -0.002 |
| NOK | 1.07 | -0.003*** | 1.94 | -0.007 | 1.38 | -0.010*** | 0.56 | 0.003 |
| PLN | 1.34 | 0.003 | 1.09 | -0.001 | 0.77 | 0.000 | 0.31 | -0.001 |
| RUB | 1.02 | 0.000 | **2.45** | -0.022** | 0.99 | -0.004 | *1.46* | -0.018** |
| SEK | 2.03 | 0.000 | 1.79 | 0.009* | 0.96 | -0.002 | 0.83 | 0.011*** |
| SGD | **4.50** | 0.000 | **2.12** | 0.017*** | 0.97 | 0.007*** | *1.16* | 0.010** |
| USD | 2.23 | 0.009*** | 1.84 | -0.011** | 0.89 | 0.001 | 0.96 | -0.012*** |

*, **, *** denote statistical significance at the 10%, 5%, and 1% level, respectively

*Source: authors' own calculations based on data from stooq.com (accessed October 21, 2024).*

The observed fluctuations in currency positions within the MST and causality networks correspond to major shifts in the global macroeconomic landscape. The weakening centrality of the USD during certain sub-periods of the pandemic may reflect the extraordinary monetary easing by the Federal Reserve and a temporary shift toward regional safe-haven assets. Similarly, the declining influence of the CNY can be attributed to China's 'Zero-COVID' policy and subsequent property market crises, which disrupted international trade structures and dampened the currency's role as a regional anchor. In contrast, the strengthening roles of the GBP and NOK during the 2022–2023 period are closely tied to the divergence in monetary policy and commodity market shocks. The GBP's increased centrality coincided with the Bank of England's early and aggressive stance against surging inflation. Meanwhile, the NOK's elevated position in the causality network following the outbreak of the war in Ukraine reflects its status as a key energy-linked currency. As European energy markets faced unprecedented volatility, the NOK became a primary channel for shock transmission, acting as a barometer for regional energy security and capital flows. These findings suggest that topological centrality is not merely a statistical artifact but a reflection of a currency's susceptibility to - and influence over - global macroeconomic shocks.

Unfortunately, our study did not confirm the result of Výrost et al. (2015) that currencies with low in-degree tend to have higher out-degree and vice-versa. In the equation *in-degree* =1.1018 – 0.1871·*out-degree*, the coefficient of the out-degree variable was statistically insignificant. Perhaps the reason for this is that the sample range was too broad and further research would require sub-period analyses.

The attempt to reduce the Granger causality networks to spanning arborescences of minimum weight (MSAs) cannot be considered successful, as the Chu-Liu/Edmonds algorithm resulted in multiple branches for each sub-period, depending on the starting vertex. The inability to construct stable and meaningful MSAs in our study constitutes a significant empirical finding in its own right. As suggested by the nature of the foreign exchange market, this outcome may stem from the absence of a single, natural 'root' currency in the global causal system. Unlike correlation networks, which can be effectively compressed into a MST, the causal structure of the FX market appears to be inherently non-hierarchical and decentralized. The failure of the MSA algorithm suggests that causal information transmission in the FX market cannot be simplified into a single directed tree without losing essential information about the complexity of the system. Furthermore, the use of p-values as edge weights, while statistically sound for identifying links, measures the significance rather than the economic strength of relationships, which may further complicate the identification of a stable arborescence. This supports the view that the FX market is a complex network of multi-directional flows rather than a simple hierarchical structure.

## 5. Conclusions

The purpose of the study was to compare two types of topological networks for the foreign exchange market: those based on correlation coefficients and those based on Granger's concept of causality. The study also aimed to demonstrate that causality networks act as a complementary framework to traditional correlation models. Accordingly, currency networks were built taking into account the strength of relationships between currency pairs and the directionality of these links.

The networks were constructed for the exchange rates of the world's 15 major currencies against the NZD, using a rolling window technique. The characteristics of the networks were analyzed.

This study set out to answer three research questions (Q1 – Q3) regarding the comparative advantages of causal versus correlation-based currency networks. Based on the empirical analysis, we formulate the following conclusions:

Ad Q1: The topological dynamics of causal networks exhibit significantly higher sensitivity to market shocks compared to correlation-based structures. While the correlation network (MST) remained relatively stable, the causality-based network responded more dynamically to the outbreak of the COVID-19 pandemic and the conflict in Ukraine, showing rapid changes in connectivity and density.

Ad Q2: The inclusion of directionality changes the identification of the market's most influential nodes. Unlike the correlation approach, which only identifies pairs of co-moving currencies, the causality-based analysis allowed us to distinguish 'source' currencies (drivers) from 'sink' currencies (followers), effectively highlighting the dominant role of the CHF during periods of instability.

Ad Q3: The causality-based approach provides substantial 'informational richness' that cannot be obtained from correlation analysis alone. It acts as a complementary framework by uncovering the hidden directional architecture of the market and identifying the specific pathways of shock transmission (contagion channels), which are essential for effective risk monitoring.

Table 4 compares the two network approaches used and is the main result of the analysis.

**Table 4.** Comparison of correlational and causal approaches to network construction

| Specification | Correlation networks | Causality networks |
|---|---|---|
| Idea | reflect the strength of relationships between pairs of currencies | reflect the strength and directionality of relationships between currency pairs |
| Preliminary assumptions | - | stationary time series |
| Consequences of the adopted distance measure | networks constructed based on Pearson's linear correlation coefficient are dependent on the assumed base currency; alternative: partial correlation coefficient | large choice of causality tests not necessarily leading to networks with identical topology |
| A way to simplify the relationship | minimum spanning tree (MST) | - reduction to graphs with edges with p-value<0.05,<br>- minimum-cost arborescence (MSA) [but here: sensitivity to initial vertex = root] |
| Density and average distance | constant density in MSTs; high average distance in crises | during periods of uncertainty density decreases, and with expansionary monetary policy increases |
| Degrees of vertices | degree of vertex determined by the total number of incoming and outgoing edges | possible identification of in/out degrees of vertices => indication of "influence-prone" and "influential" nodes |
| Further applications | allow the construction of a hierarchical structure of the market | helpful in studying the dynamic propagation of shocks in the currency system |

*Source: author's own investigation.*

The foreign exchange market is a complex system. Its complexity is accompanied by specific interdependence. In this article, we have proposed to quantify this interdependence using correlation networks and Granger causality networks. Correlation networks, and especially minimum spanning trees, provide broad insights into the linkages between currencies, while Granger causality networks capture the complex web of statistical relations between them. The linkages between currencies during the period under review were highly dynamic and changed over time depending on market and political conditions. The use of a wide range of tools to assess the topology of the networks allowed a better understanding of the phenomena taking place in the foreign exchange market.

Despite the insights provided by this comparative analysis, certain methodological limitations should be acknowledged. First, the results remain conditional on the chosen reference currency (numéraire). While the choice of NZD was a strategic decision to minimize artificial centralization, future research could employ numéraire-independent approaches. These might include valuing individual currencies against a weighted basket of currencies or utilizing partial correlations to further mitigate common-factor bias.

Second, the use of linear Granger causality focuses on dependencies in the conditional mean. While this provides a robust and interpretable baseline for identifying directional information flows, it may not capture nonlinear dynamics, threshold effects, or dependencies in higher moments of the distribution (e.g. volatility spillovers and tail behavior), which are characteristic of foreign exchange markets during periods of extreme stress. With nonlinear extensions of Granger causality, a higher degree of interconnectedness between currencies could potentially be uncovered. Consequently, the causality networks reported here should be interpreted as representations of local, short-term predictive relationships.

Finally, while we adopted a uniform lag length of $k$=1 - motivated by the high informational efficiency of global FX markets and statistical parsimony - interactions in less liquid markets or during specific regime changes might materialize with longer delays. Although our rolling window approach explicitly accounts for local shifts in market regimes (such as the pandemic or the conflict in Ukraine), future studies could explore the use of frequency-domain causality or time-varying lag structures to further refine the map of information transmission in the global financial system.

Treating the constructed Granger causality networks as a starting point for further studies, we intend to focus on tracking the evolution of vertex degrees for specific currencies and drawing conclusions in this regard.

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